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DUAL BOUNDS VARIATIONAL FORMULATION OF SKIN EFFECT PROBLEMS

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SUMMARY

Accurate loss calculation of transmission lines is an important topic in monolithic microwave integrated circuits (MMICs). This paper describes a general variational approach for calculating dual bounds of the interesting circuit parameters. Using the dual bounds approach, the computational expense can be reduced drastically; the accuracy of the solution for the interesting circuit parameters R,L is guaranteed by the corresponding upper and lower bounds. Combined with an improved classical full-wave analysis, the method presented here is a good tool for the loss calculation due to the skin effect in microstrip like structures.

INTRODUCTION

The rigorous description of skin effect losses in strip lines becomes more and more important, because in monolithic microwave integrated circuits the metallization thickness is of the order of the skin-depth even at high frequencies and therefore the up to now used incremental inductance rule formulas /1/ must be critically revised. It is especially important to deal with a real two-dimensional current displacement in the cross section of the strip, if the width and the thickness of the structure are of the same order.

A perturbation method based on the well known spectral domain approach for calculating strip line losses was presented on the MTT-S Symposium in 1986 /2/. Several assumptions were made to include the influence of the finite metallization thickness in the spectral domain formulation. As a consequence, this method cannot be regarded as an exact method any longer. Furthermore, the losses caused by the current distribution in the small sides of the strip are not considered in this one-dimensional approximation. In addition, the main formula (1) in /2/ describing the current distribution is not correct.

Therefore the problem of calculating the influence of a two dimensional current displacement in the cross section of the conductor shall be discussed again. A general method for calculating the skin effect resistance and the inner inductance of strips with rectangular cross section and finally its application to microstrip lines shall be presented. Only one assumption is needed: The knowledge of the electromagnetic field distribution on an arbitrary curve surrounding the strip is necessary for formulating the boundary value problem.

Hammond /3/ remarked in a fundamental investigation in 1980, that the variational approach is a very powerful method for the solution of skin effect problems. He pointed out, that the dual formulation of the electromagnetic field leads to the specification of dual bounds for the circuit parameters. The method presented here is a generalization based on this idea, including the field distribution outside the conducting material, which is essential in most boundary value problems. Combined with an improved full-wave analysis /6/ of the microstrip line, values for the losses in microstrip structures including edge effects can be calculated.

THE VARIATIONAL APPROACH

In the following the quasi-stationary cylindrical electromagnetic boundary value problem shown in Fig.1 shall be considered. Ω_κ is the cross section of the conducting area described by the conductivity κ and a permeability $\mu=\mu_0$. The boundary

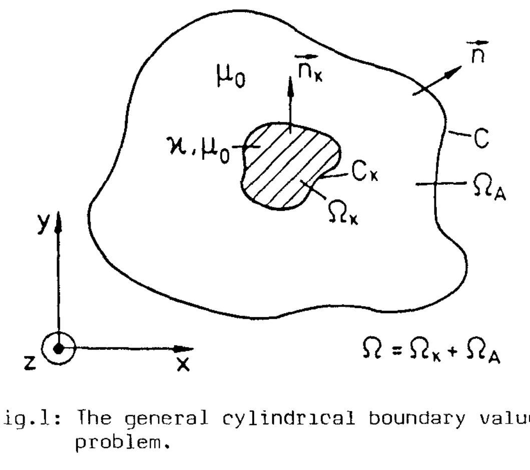

Fig.1: The general cylindrical boundary value problem.

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curve C_K surrounds the counductor, C is the boundary of the total considered field region. The current density in the conducting area shall exhibit only a z-component.

A general variational method for calculating skin effect problems has been described by the authors lately /5/. As has been shown in this publication, the electromagnetic field problem defined in Fig.1 can be solved in two different ways: In the first method the electromagnetic field in both regions (conducting subregion and region outside the conductor) is calculated using the vector potential $\vec{A}$; in the second method the field inside the conductor is derived from the magnetic induction $\vec{B}$, whereas the field in the outer region is described by the vector potential again.

As can be shown, both formulations of the problem lead to dual variational formulations for the computation of the electromagnetic field inside the exterior curve C (Fig.1). Provided that such formulations are found, the prescribed boundary values on the exterior curve C must be natural and essential for the dual field representations, respectively. If e.g. the electromagnetic field in Ω is described in terms of the magnetic potential , a specified boundary distribution of $\vec{A}$ itself is an essential boundary condition, whereas prescribed values of the first derivative in normal direction, i.e. the tangential magnetic induction $\vec{B}_t$, imply a natural boundary condition.

Principally the dual variational formulation for a given boundary condition can be established by the dual variables $\vec{A}$ and $\vec{B}$. Although this concept is a serious mathematical basis for the variational technique, the numerical realization based on $\vec{B}$ in the total field area leads to some problems concerning the continuity conditions on the inner boundary C_K, because $\vec{B}$ has no continuous derivation with respect to the normal on C_K. Therefore it is more advantageous to use the dual variational concept only for the field representation inside the conducting area; the outer region can be described by the magnetic vector potential again. According to this concept, the displacement current can be neglected even in the outer region.

NUMERICAL VERIFICATION

The described dual variational concept which is capable to analyze two-dimensional current distributions in conducting media must be verified. For this purpose the one-dimensional problem shown in Fig.2, which can be solved exactly, shall be used. Two boundary conditions may be assumed on the outer curve C: $A_z=0$, or $B_x=B_0=\text{const.}$.

In the case that the vector potential $\vec{A}$ is used to describe the electromagnetic field in both field regions, geometrical expansion functions for the complex vector potential $A_z=A_z'+jA_z''$ as shown below are used:

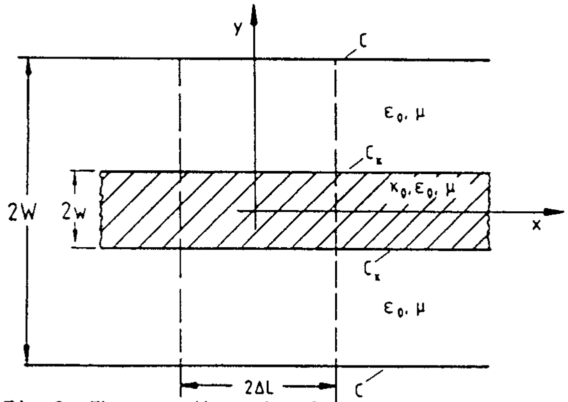

Fig.2: The one-dimensional boundary value problem for the verification of the method.

In the airfilled region:

$$A_z'(y) = \mu I \sum_{i=1}^{N_a} \gamma_i \cos\left((2i-1) \frac{\pi}{2W} y\right) ,$$

$$A_z''(y) = \mu I \sum_{i=1}^{N_a} \delta_i \cos\left((2i-1) \frac{\pi}{2W} y\right) , \qquad (1)$$

In the conducting region:

$$A_z'(y) = \mu I \sum_{i=1}^{N_i} \alpha_i \cos\left((2i-1) \frac{\pi}{2w} y\right) + \mu I \sum_{i=1}^{N_a} \gamma_i \cos\left((2i-1) \frac{\pi}{2W} y\right) ,$$

$$A_z''(y) = \mu I \sum_{i=1}^{N_i} \beta_i \cos\left((2i-1) \frac{\pi}{2w} y\right) + \mu I \sum_{i=1}^{N_a} \delta_i \cos\left((2i-1) \frac{\pi}{2W} y\right) . \qquad (2)$$

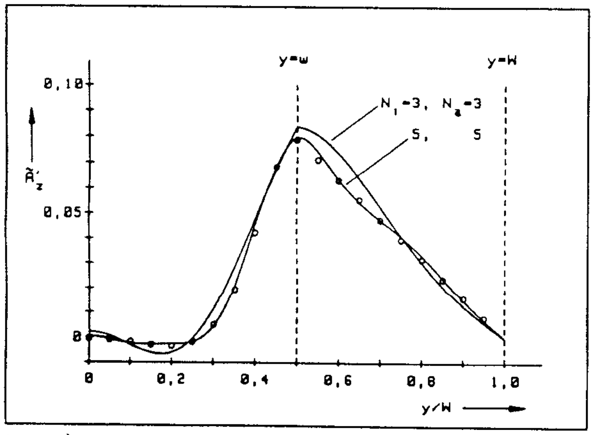

Fig.3a): Field distribution of the real part of the magnetic vector potential for different cut-off indices N_i, N_a. w/W=0.5 (see Fig.2). ooo exact solution. $\tilde{A}_z=A_z(j\omega\kappa 4w\Delta L/I)$.

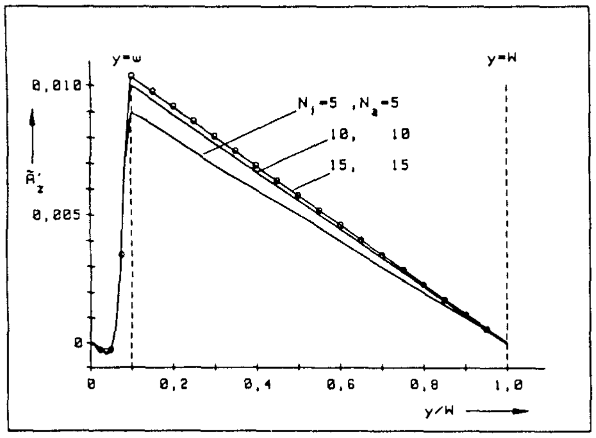

Fig.3b): Field distribution of the real part of the magnetic vector potential for different cut-off indices N_i, N_a. w/W=0.1 (see Fig.2). ooo exact solution. $\tilde{A}_z = A_z(j\omega\kappa 4w\Delta L/I)$.

Using this field expansions and the theoretical background given in /5/, the field distributions shown in Fig.3a) and Fig.3b) with the upper cut-off indices N_i and N_a as parameters can be found and compared with the exact solution of the problem. As can be seen, even the field distributions, which are local values, are quite well approximated by the variational approach, provided the cut-off indices are larger than 5.

In a similar form the magnetic induction $\vec{B}$ and the vector potential $\vec{A}$ can be formulated in the dual method.

In the airfilled region:

$$A_z'(y) = \mu I \sum_{i=1}^{N_a} \gamma_i \cos\left((2i-1)\frac{\pi}{2W} y\right) ,$$

$$A_z''(y) = \mu I \sum_{i=1}^{N_a} \delta_i \cos\left((2i-1)\frac{\pi}{2W} y\right) , \qquad (3)$$

and in the conductor region:

$$B_x'(y) = \mu I \sum_{i=1}^{N_i} \alpha_i \left(\frac{i\pi}{2w}\right) \sin\left(i\frac{\pi}{2w} y\right) ,$$

$$B_x''(y) = \mu I \sum_{i=1}^{N_i} \beta_i \left(\frac{i\pi}{2w}\right) \sin\left(i\frac{\pi}{2w} y\right) . \qquad (4)$$

The advantage of this dual variational formulation becomes obvious, if the convergence behaviour of the global circuit parameters R and L_i is discussed. In Fig.4 and Fig.5 complementary results for the frequency dependence of these parameters are given. It can be seen, that the confidence in the solution is improved by the specification of upper and lower bounds; furthermore a simple arithmetic average value reduces the convergence error remarkable. For this conclusion to be true, it must be required, that the two dual methods use the same functional approximation system.

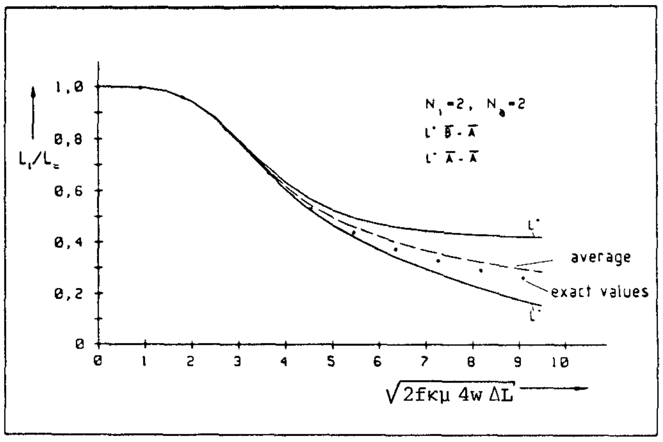

Fig.4: Dual bounds for the normalized inner inductance L_i in dependence on a normalized frequency. $L_=$ is the dc-value of the inductance. W=3w (see Fig.2).

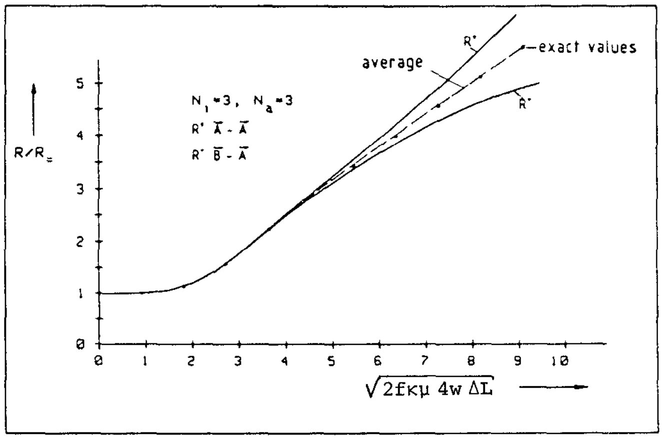

Fig.5: Dual bounds for the normalized resistance R in dependence on a normalized frequency. $R_=$ is the dc-value of the resistance. W=3w (see Fig.2).

In a first step for the application of this variational concept to a two-dimensional poblem, the computation of the frequency dependent resistance of a free single strip conductor with rectangular cross section was performed. In table 1 some specific values for the normalized resistance $R/R_=$, where $R_=$ is the dc-value of the resistance, are given in dependence on the upper cut-off index of the field expansion series. It can be seen, that also in this example the convergence is guaranteed by the specification of the upper and the lower bounds.

The described method for the first time delivers a possibility to calculate the resistance of a strip or thin film resistor, taking into account the two-dimensional current density distribution in the rectangular cross section. This method therefore can be used to compute the Q-factor of e.g. lumped elements very accurately.

M_i	B-A	F_{rel}	A-A	F_{rel}	Average	F_{rel}
2	2.1193	-37.3%	3.6681	+8.5%	2.8934	-14.4%
3	2.9748	-12.0%	3.5938	+6.3%	3.2843	-2.8%
4	3.2612	-3.5%	3.5680	+5.5%	3.4146	+1.0%
5	3.2699	-3.2%	3.4953	+3.2%	3.3826	+.007%
6	3.3064	-2.2%	3.4580	+2.2%	3.3822	+.006%

Table 1: Dual bounds for the normalized resistance $R/R_=$ of a strip conductor with rectangular cross section in dependence on the cut-off index M_i; $(2f\kappa\mu 4wt)^{1/2}=8.35$. 2w=strip width, 2t=strip thickness. w=t.

THE MICROSTRIP LOSSES

The exact calculation of losses caused by the skin effect in the conducting materials of a microstrip line with finite metallization thickness is very complicated, because 1) most of the efficient methods for calculating the electromagnetic fields of microstrip structures (e.g. the spectral domain method) must assume zero thickness of the metallization, and 2) because the introduction of the conducting areas into the field region involves the necessity to consider material parameters which are of highly different magnitude, and 3) because the field equations become complex, if the skin effect is considered. Therefore an approximating technique is used here; but in contrary to other approximations (e.g. /1/) it leads to a real two-dimensional current density distribution inside the conducting strip.

Provided that the exact field distribution of the magnetic induction $\vec{B}$ can be calculated on the strip surfaces and on the surface of the ground metallization, the corresponding field distribution inside the conducting material can be found easily. A full-wave approach based on the classical eigenfunction technique (mode matching technique) leads to numerical complications, if the field in the envirement of the four metal edges of the strip shall be computed (edge effects). To overcome this difficulty, modifications of this technique are needed. Following Bögelsack et al. /6/, the introduction of a projection method into the formulation of the continuity condition leads to a remarkable improvement. In this case the field distribution on a curve C, surrounding closely the metal strip, can be given explicitely. Applying the perturbation method, the field inside the domain enclosed by C can be calculated using dual variational principles, as described above.

Fig.6 shows the attenuation of a microstrip line calculated as described above, compared to those values, which have been computed with the incremental induction rule /1/. It can be seen from the figures, that the classical incremental induction rule is not able to describe the sharp increase of the attenuation with decreasing metallization thickness. For high values of the metallization thickness both methods deliver nearly the same results.

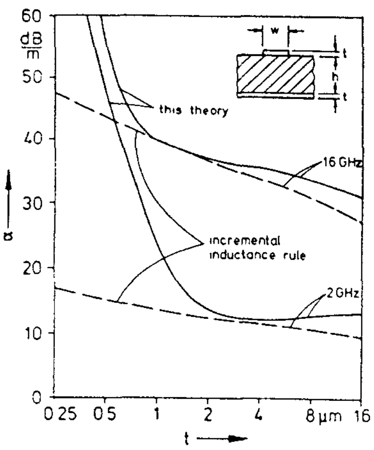

Fig.6: Comparison of the microstrip attenuation calculated with the incremental induction rule and with the variational approach. w=72.4 µm, h=100 µm, ε_r=12.9.

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